

# Scaling Law for Boundary Layer Inner Eyewall Pumping in Concentric Eyewalls

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# Introduction

- ERC is important because it can lead to increases in both storm size and integrated kinetic energy. (Sitkowski et al., 2011)
- Concentric eyewalls (CE) duration: from the formation of an outer eyewall to the complete decay of an inner eyewall.
  - Short-lived CE: The CE of TC Lekima (2001) and TC Andrew (1992) sustained for only 6 hours from radar observations. (Kuo et al., 2004; Willoughby and Black, 1996)
  - Average ERC durations from aircraft data: 36 hours (Sitkowski et al., 2011)
  - Average ERC durations from microwave satellite data: 17.5 hours
- CEs with durations longer than 20 hr tends to have a larger moat width and larger outer eyewall width.
- The western North Pacific (WNP) has more long-lived CEs than in Atlantic (ATL) and in the eastern North Pacific (ENP). (23% for WNP, but 5% for ATL)

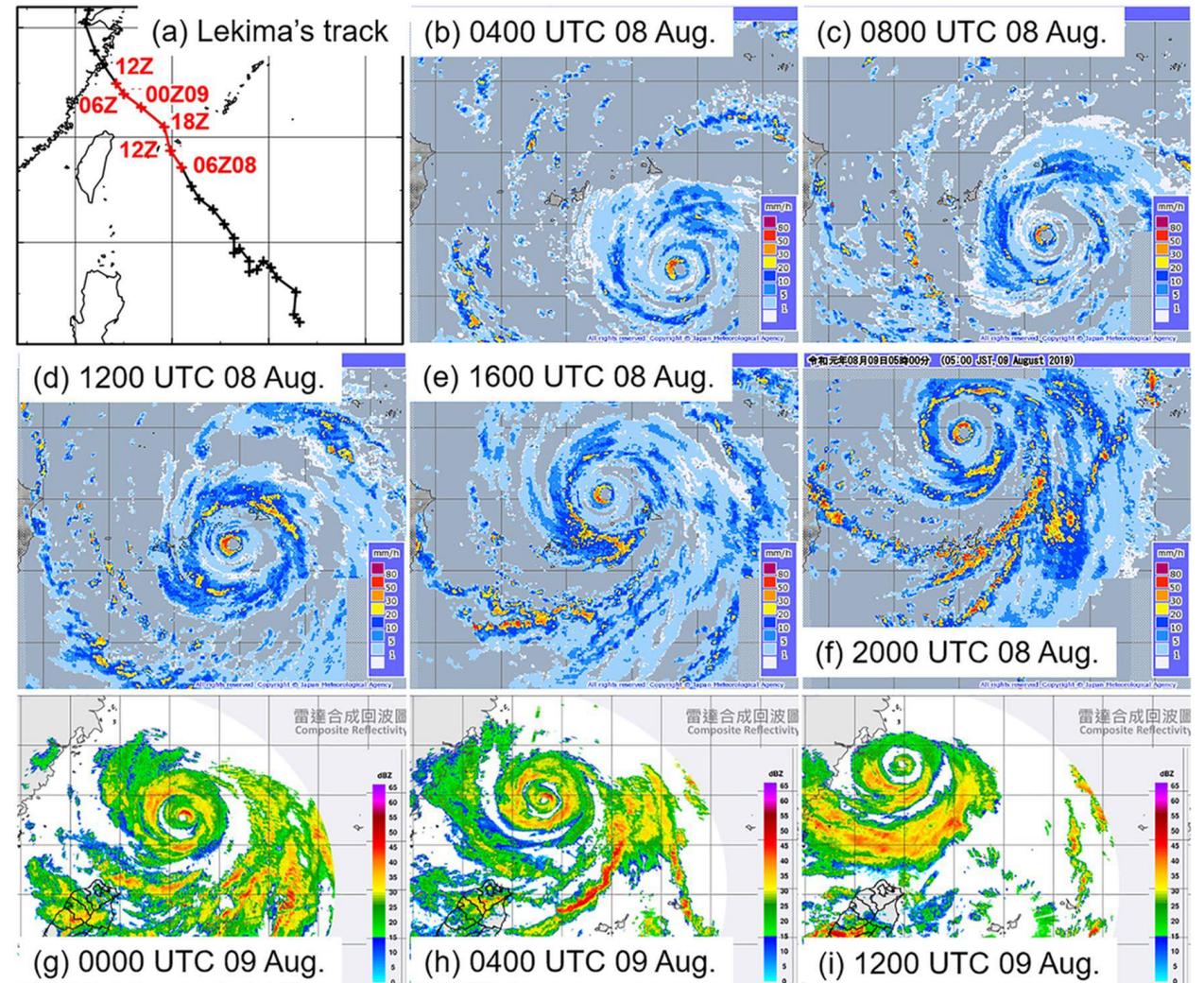
(Yang et al., 2021)

# Features of the long-lived CEs

- TC Soulik (2013) had 2 long-lived CE episodes. The first one sustained for 25 hours, and the second one sustained for 34 hours. A large moat size and outer eyewall width were present in both CE periods.

(Yang et al., 2013)

- TC Lekima (2019):
  - more than 30 hours CE duration
  - Traveling distance about 600 km
  - Moat size: 40 km
  - Inner eyewall radius: 10 km
  - Strong convections in the inner eyewall



# Benefits of large moat size

- Outer eyewall needs more time to contract in a wider distance.
- Barotropic instability, which causes the inner eyewall spindown but the outer eyewall spinup, grow slowly in a wide moat with low radial vorticity gradient.  
(Kossin et al., 2000)
- Partial blockage of moisture supply due to asymmetry may prolong the CE structure. A wide moat size allows more moisture to be pick up from the sea surface.  
(Tsujino et al., 2017)
- The subsidence warming, which is unfavorable for convections, in the moat is enhance by the outer eyewall convection in the Sawyer-Eliassen diagnoses. A large moat size can weaken this effect.  
(Rozoff et al., 2008)

# Introduction

- What may control the size of CE TCs?
  - A vortex-skirt TC with sufficient strength favor the formation of CE storm with large moat. (Kuo et al., 2008)
  - The size of long-lived CE storms is larger in warm and normal episodes of the ENSO than that in cold episodes of the ENSO in WNP. (Yang et al., 2015)
- BL dynamic is important to TC intensity.
  - From the aircraft observations, the radial inflow decreased from 22 to 0 m/s in a few kilometers in Hurricane Hugo (1990). Williams et al. (2013) used a slab boundary layer (SBL) model to show the nonlinear radial advection produce a shock-like structure. It was also observed and can produce large vorticity.
  - The shock-like structure was reproduced in the 500-m resolution simulation for TC Haiyan (2013). This structure can produce over 200 PVU of PV tower. (Tsujino and Kuo, 2020)
  - The aircraft observations of Hurricane Patricia (2015) also reported that hundreds of PVU of PV tower existed during RI period. (Martinez et al., 2019)

# Introduction

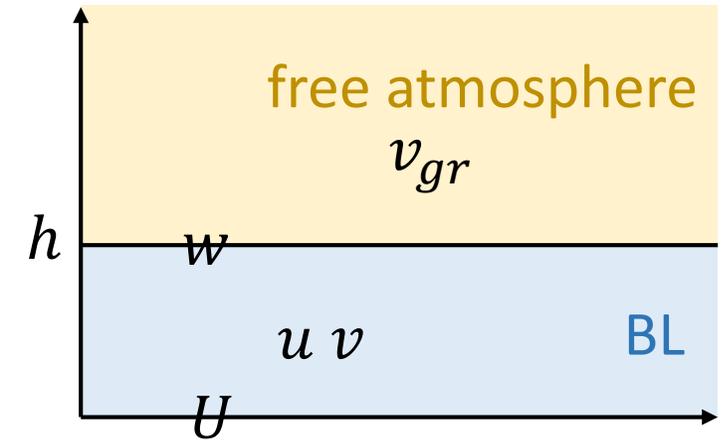
- The slab boundary layer model (SBL model):
  - a model which considers BL only
  - low degrees of freedom (simple)
  - can capture the nonlinear radial advection effect in a narrow region (e.g., moat)
- This paper try to find:
  - The relationship among maximum inner eyewall updraft (IEP), moat width, radius of inner eyewall, and maximum wind speed.
  - The positively contribution of a large moat to the long-lived CE TCs.

# Slab-Boundary Layer Model (1-D)

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - w^- \frac{u}{h} + \left( f + \frac{v + v_{gr}}{r} \right) (v - v_{gr}) - C_D U \frac{u}{h} + \kappa \frac{\partial}{\partial r} \left( \frac{\partial r u}{r \partial r} \right)$$

$$\frac{\partial v}{\partial t} = - \left( f + \frac{\partial r u}{r \partial r} \right) u - w^- \left( \frac{v - v_{gr}}{h} \right) - C_D U \frac{v}{h} + \kappa \frac{\partial}{\partial r} \left( \frac{\partial r v}{r \partial r} \right)$$

$$w = -h \frac{\partial r u}{r \partial r}$$



where:

$w^- = (|w| - w)/2$  Ekman suction

$v_{gr}$  gradient wind (represent PGF above BL)

$U = 0.78 \sqrt{u^2 + v^2}$  10 m height wind

$$C_D = 10^{-3} \times \begin{cases} 2.70/U + 0.142 + 0.0764U & \text{if } U \leq 25 \text{ m/s} \\ 2.16 + 0.5406 \left[ 1 - \exp\left(-\frac{U - 25}{7.5}\right) \right] & \text{if } U \geq 25 \text{ m/s} \end{cases}$$

Configuration and parameters:

$$dr = 100 \text{ m}$$

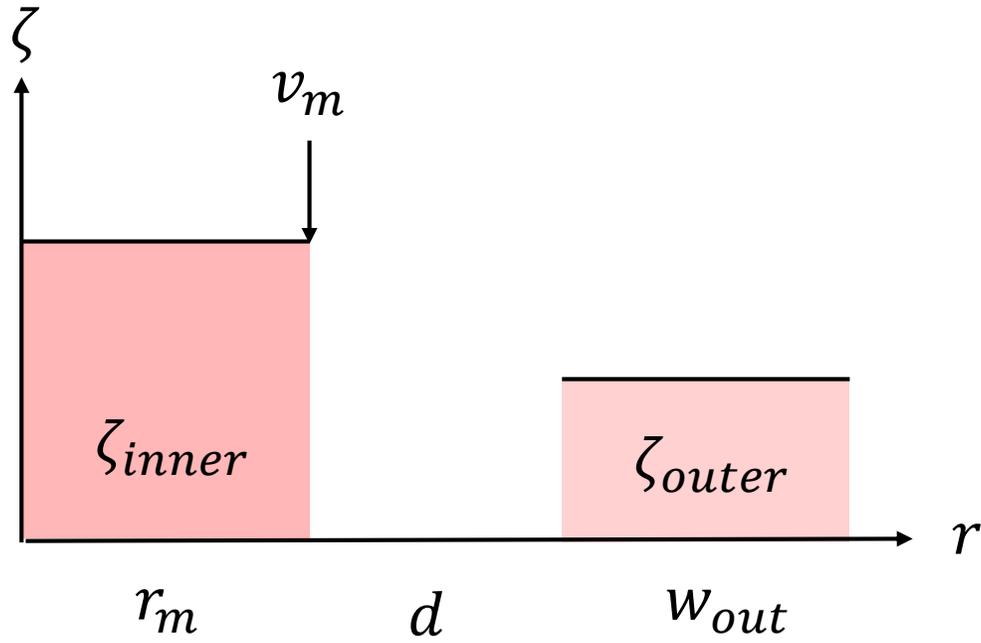
$$\kappa = 1500 \text{ (m}^2/\text{s)}$$

$$h = 1000 \text{ m}$$

$$f = 5 \times 10^{-5} \text{ s}^{-1}$$

B.C.:  $u = 0$  at  $r = 0$  and  $\frac{\partial r u}{\partial r} = 0$  at  $r \rightarrow \infty$

# Experiment Design



$r_m = 10, 20, 30, 40, 50$  (km)

$v_m = 10, 20, 30, 40, 50, 60$  (km)

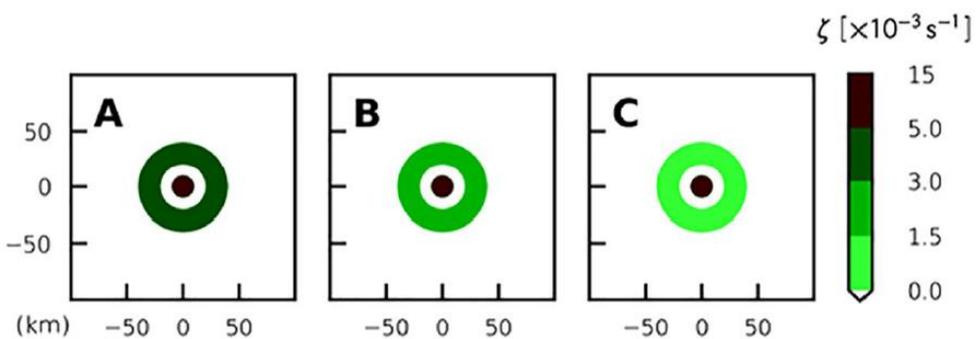
$d = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$  (km)

**Table 1**  
*Parameters for the Experiments*

Figure	Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
Figure 2	A					4
	B	10	60	10	20	2
	C					1
Figure 3	A				30	
	B	10	60	10	20	2
	C				10	
Figure 4	A			10		
	B	10	60	20	20	2
	C			—	—	—
Figure 5	A		30			
	B	10	40	10	20	2
	C		60			
Figure 6	A	20				
	B	15	60	10	20	2
	C	10				

# Results

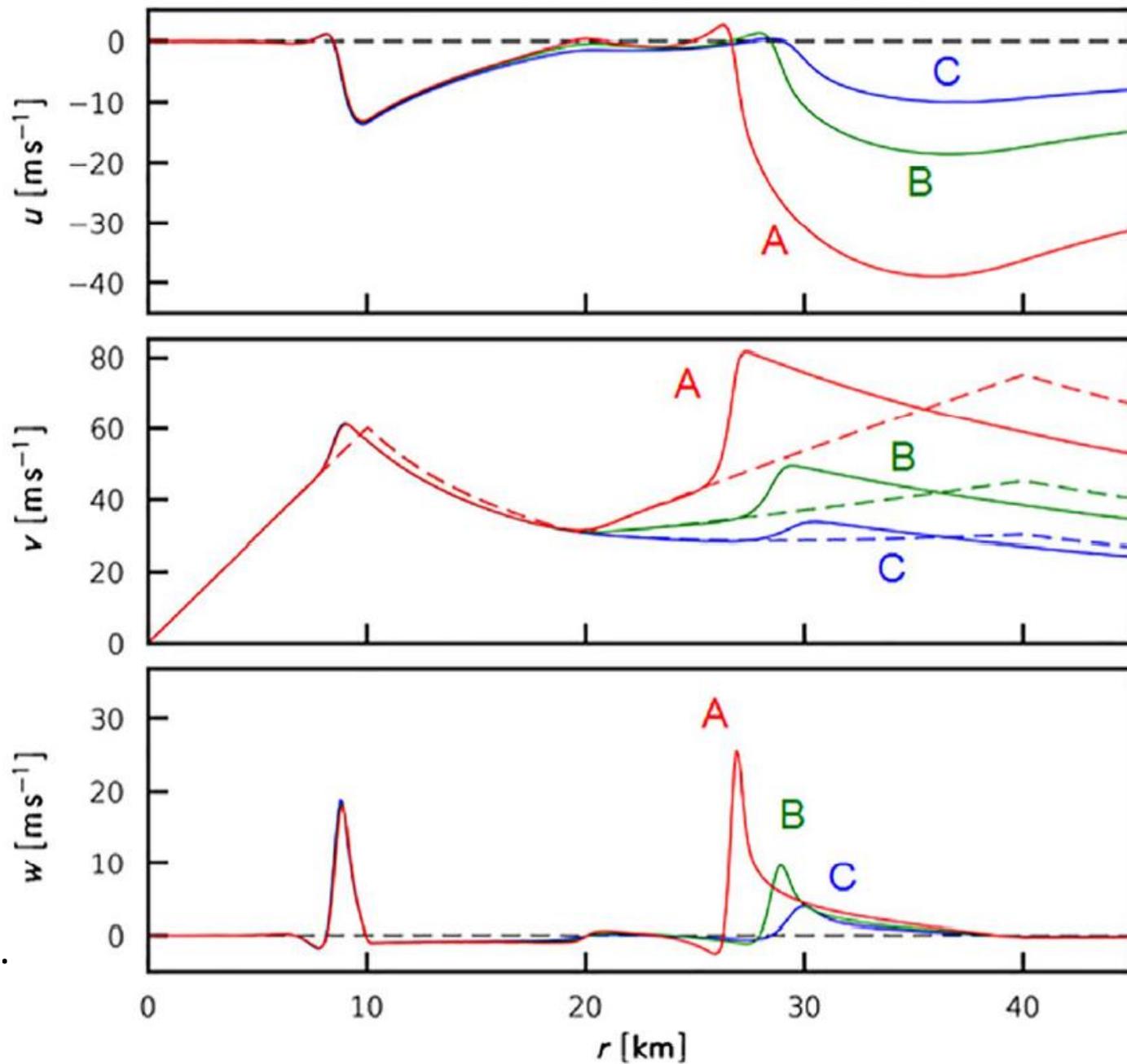
Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A					4
B	10	60	10	20	2
C					1



Stronger  $\zeta_{outer}$ :

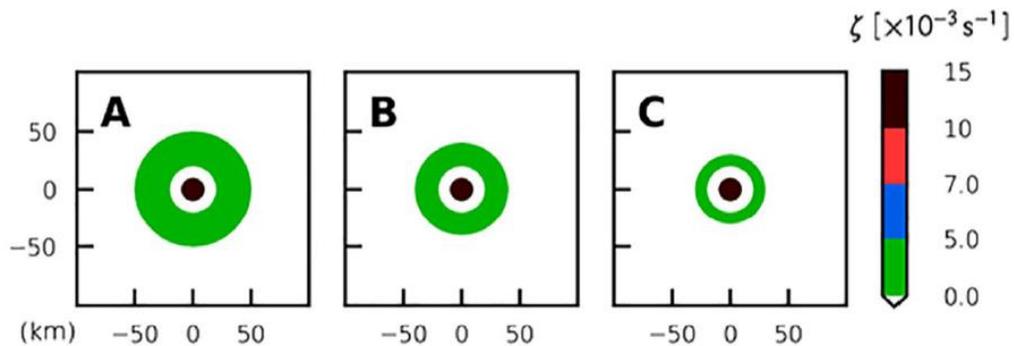
stronger BL pumping at the outer eyewall  
the outer eyewall is pushed more inward

The IEPs are similar in magnitude and location.



# Results

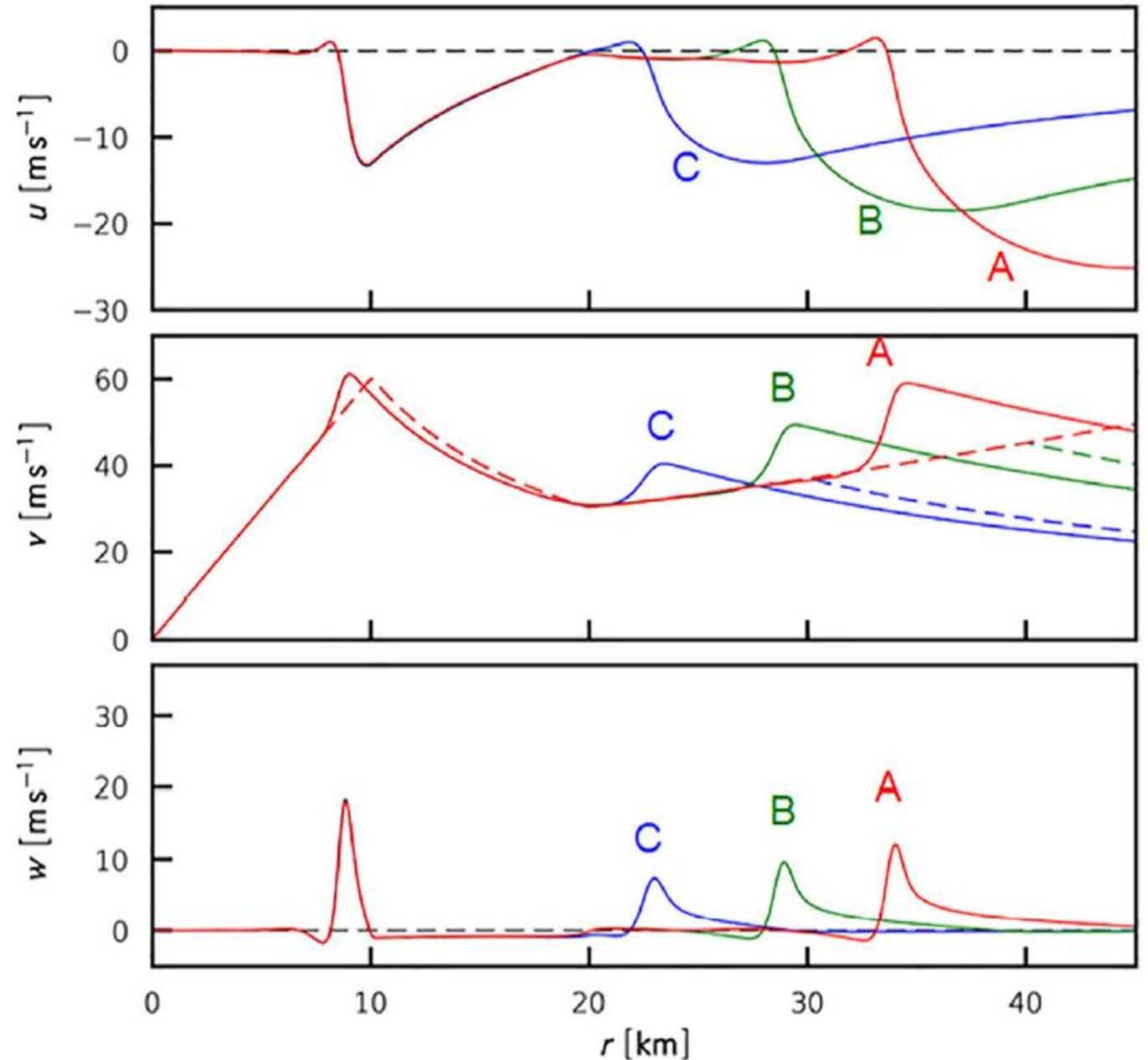
Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A				30	
B	10	60	10	20	2
C				10	



Larger  $w_{out}$ :

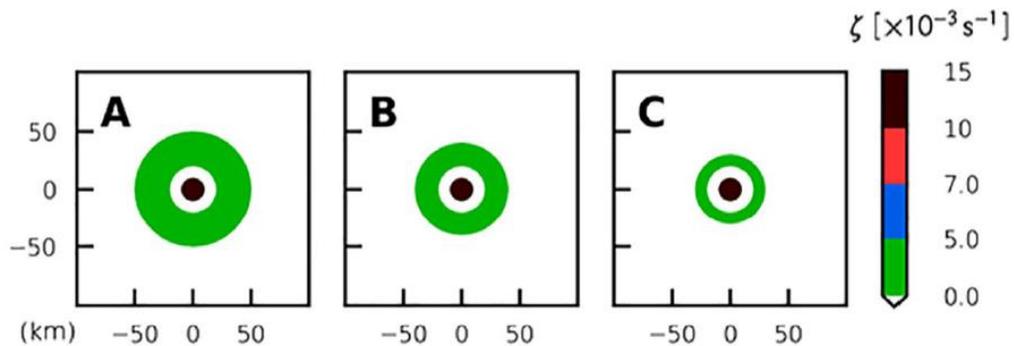
stronger outer eyewall vertical velocity  
the outer eyewall is wider

The IEPs are similar in magnitude and location.



# Results

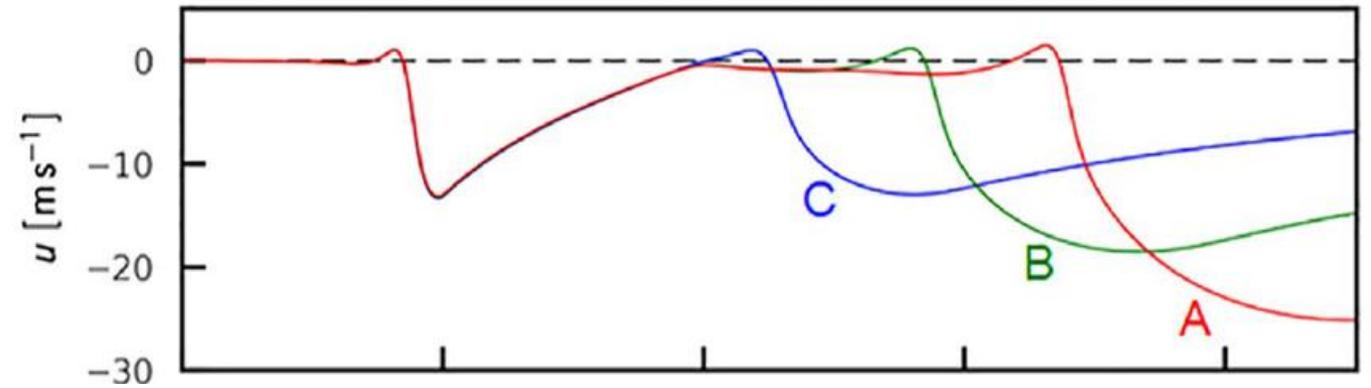
Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A				30	
B	10	60	10	20	2
C				10	



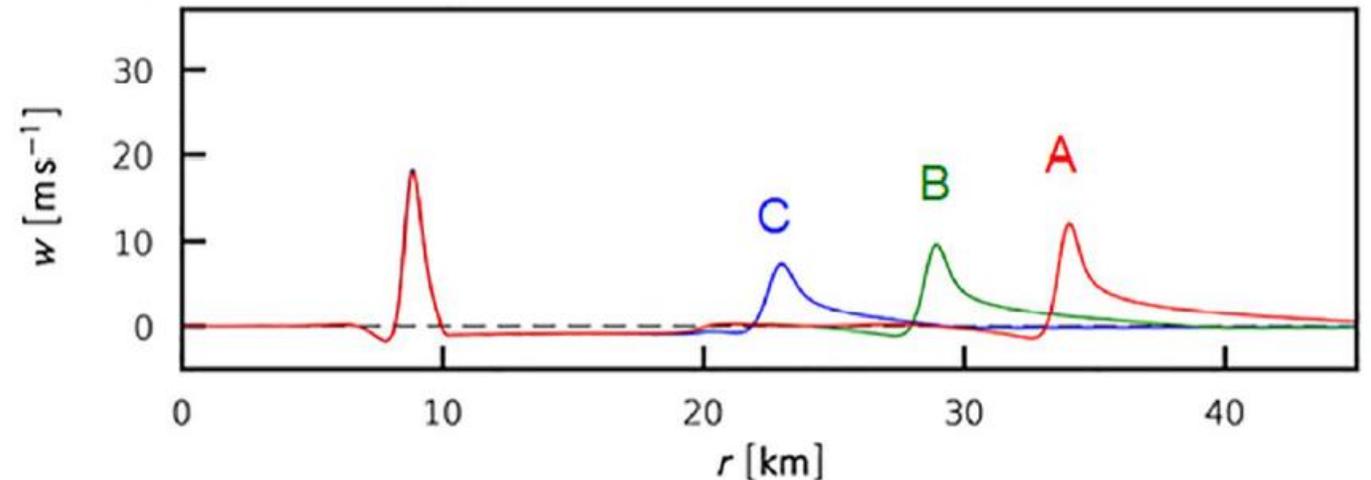
Larger  $w_{out}$ :

stronger outer eyewall vertical velocity  
the outer eyewall is wider

The IEPs are similar in magnitude and location.

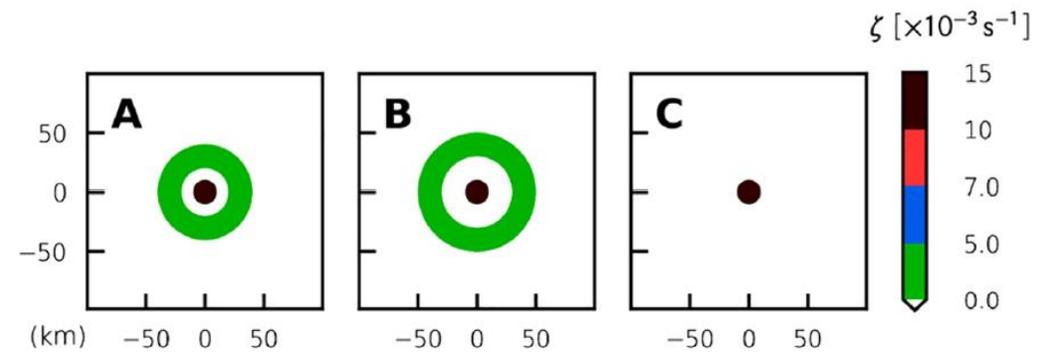


The inflow turns into updraft at the outer eyewall. Due to the PGF, the inflow reaccelerates inside the outer eyewall, which is the region affected by  $r_m$ ,  $v_m$ , and  $d$  only.

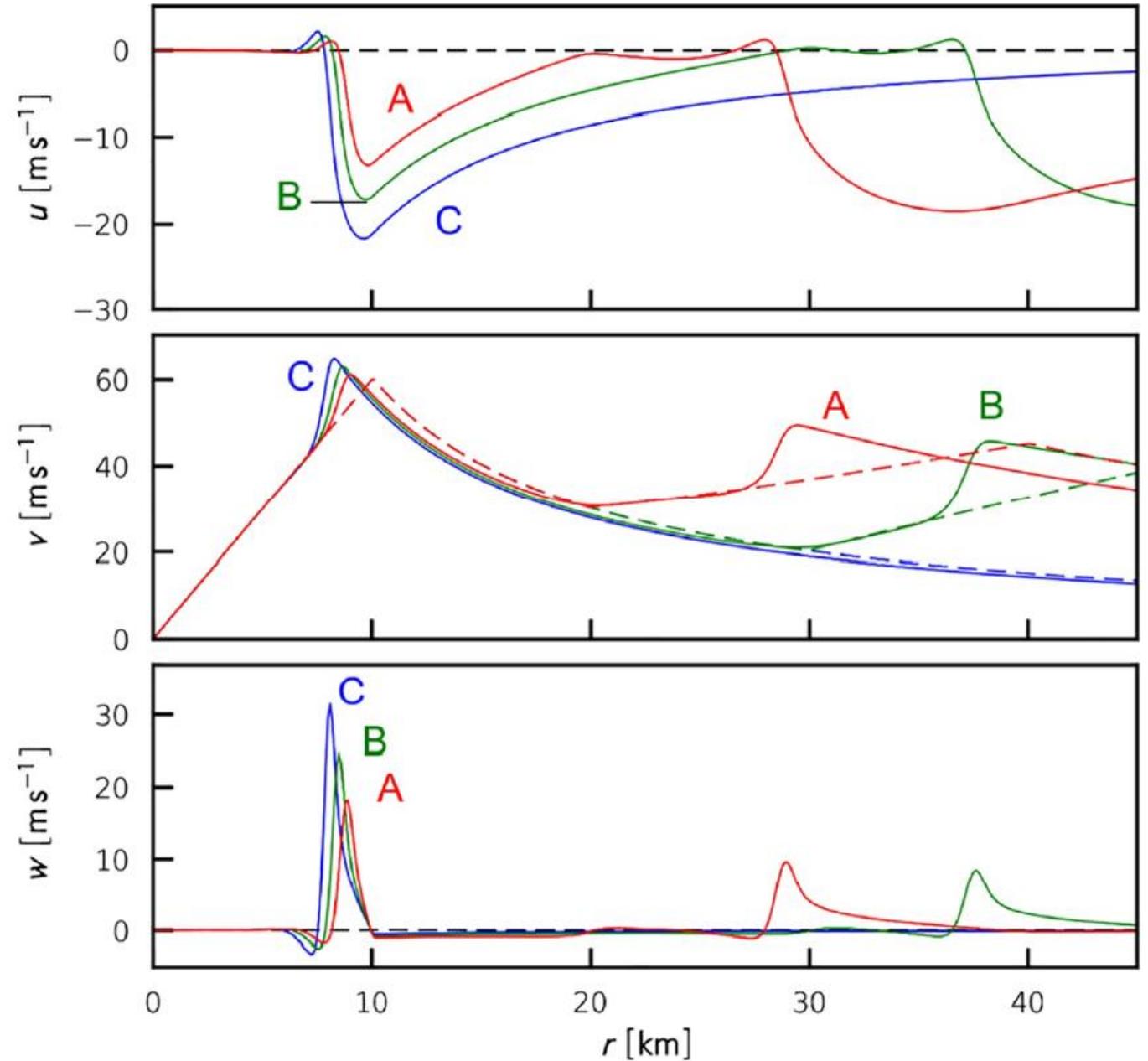


# Results

Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A			10		
B	10	60	20	20	2
C			—	—	—

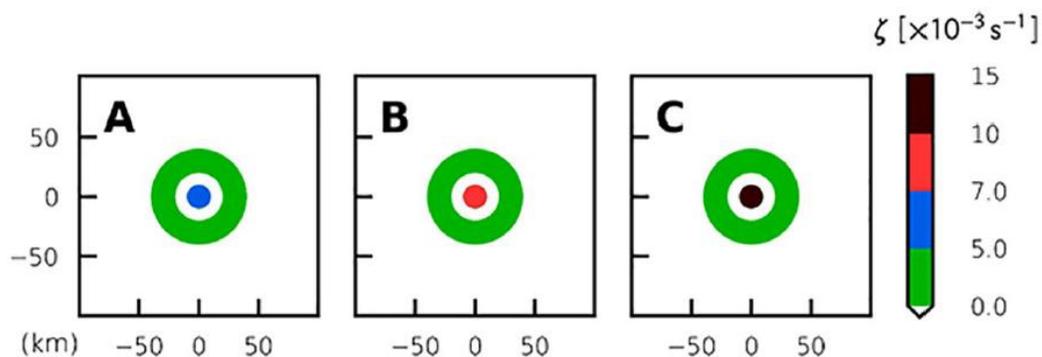


Super-gradient wind exists at RMW.  
 Shock-like structure in radial wind at RMW.  
 strength of inflow and IEP:  $C > B > A$   
 Wider moat provides space for inflow  
 reacceleration.

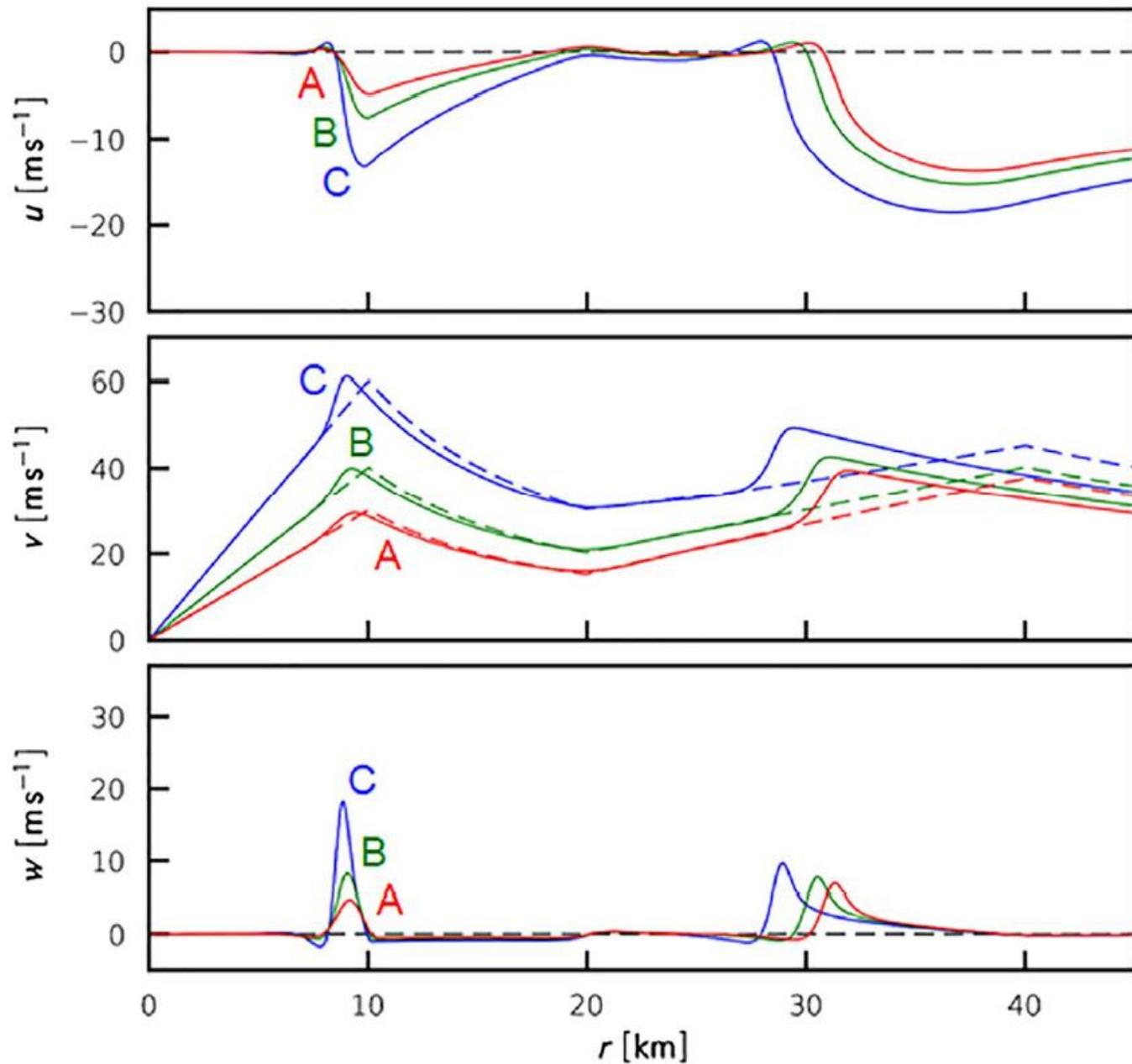


# Results

Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A		30			
B	10	40	10	20	2
C		60			

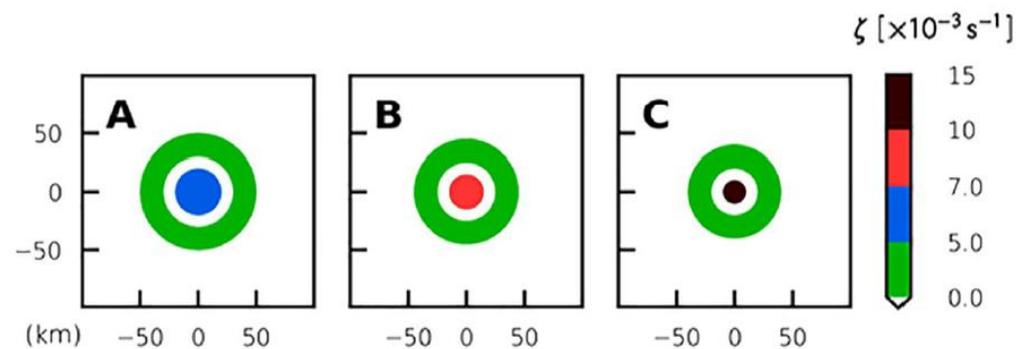


Stronger tangential wind in inner eyewall:  
Stronger inner eyewall updraft



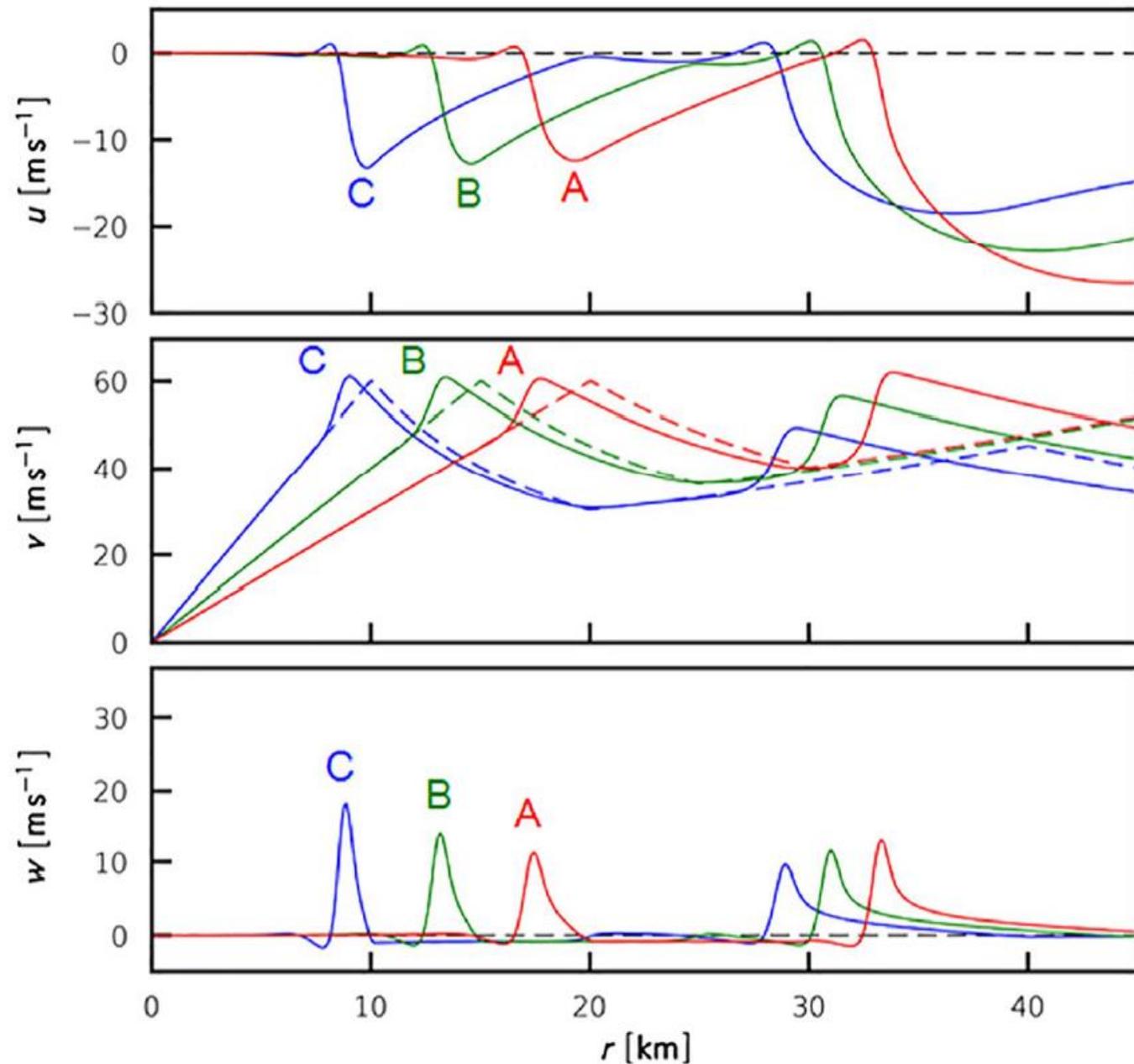
# Results

Experiments	$r_m$ (km)	$v_m$ (m s <sup>-1</sup> )	$d$ (km)	$w_{out}$ (km)	$\zeta_{outer}$ (10 <sup>-3</sup> s <sup>-1</sup> )
A	20				
B	15	60	10	20	2
C	10				



Wider inner eyewall:  
Weaker inner eyewall updraft

*IEP* increases with:  
increase  $d$ ,  $v_m$ ,  $\zeta_{inner}$ , decrease  $r_m$



# Simulation result

Experiments:

$$r_m = 10, 20, 30, 40, 50 \text{ (km)}$$

$$v_m = 10, 20, 30, 40, 50, 60 \text{ (km)}$$

$$d = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 \text{ (km)}$$

The moat size becomes larger, the inflow becomes larger. But the inflow is saturated when the moat size is large enough. (PGF is too small)

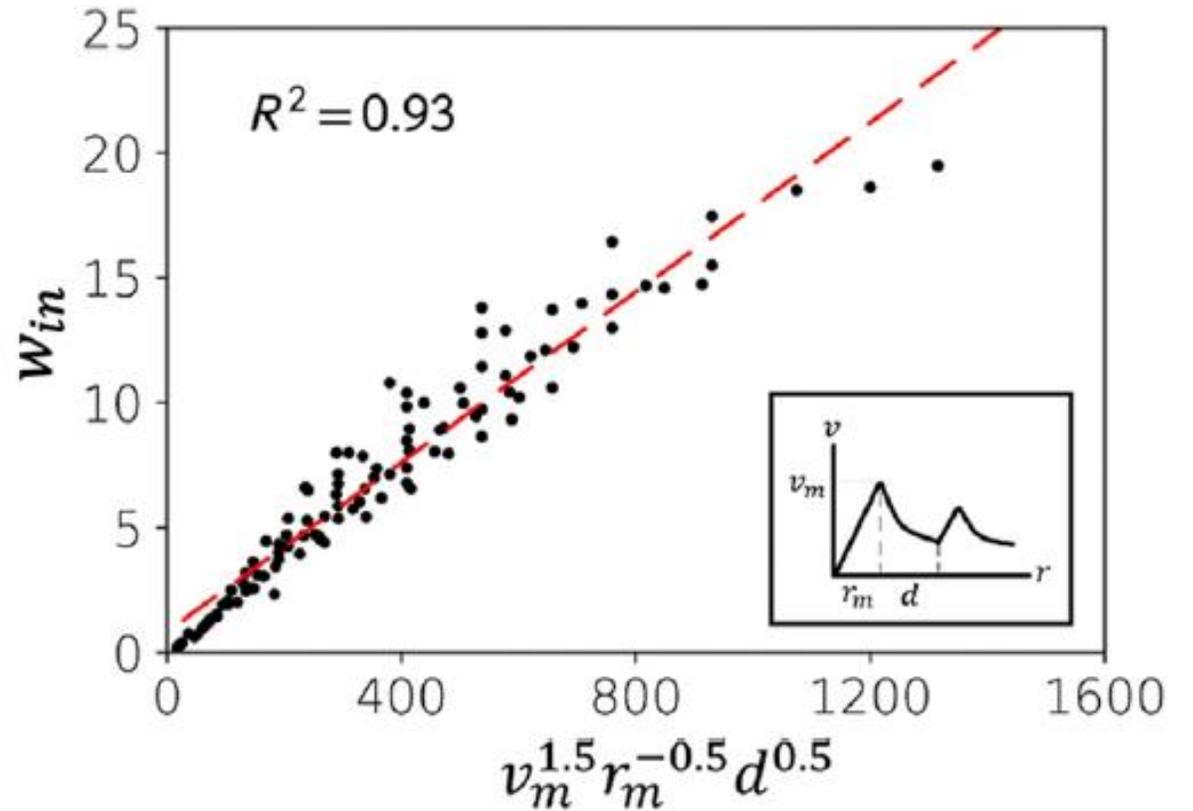
Constrain:  $\sqrt{v_m/d} > 0.03$

Relationship:

$$w_{in} \sim v_m^{1.5} r_m^{-0.5} d^{0.5} \sim \sqrt{\zeta d}$$

where

$$\zeta = 2v_m r_m^{-1}$$



regression line:  $w_{in} = 0.017 \times v_m^{1.5} r_m^{-0.5} d^{0.5} + 0.85$

$$R^2 = 0.93$$

Only experiments satisfy  $\sqrt{v_m/d} > 0.03$  is considered

# Simulation result

$$w^* = w_{in}/v_m$$

$$u^* = u/v_m$$

$$d^* = \zeta d/c$$

By *work = force × distance*:

$$d^* \sim u^{*2}$$

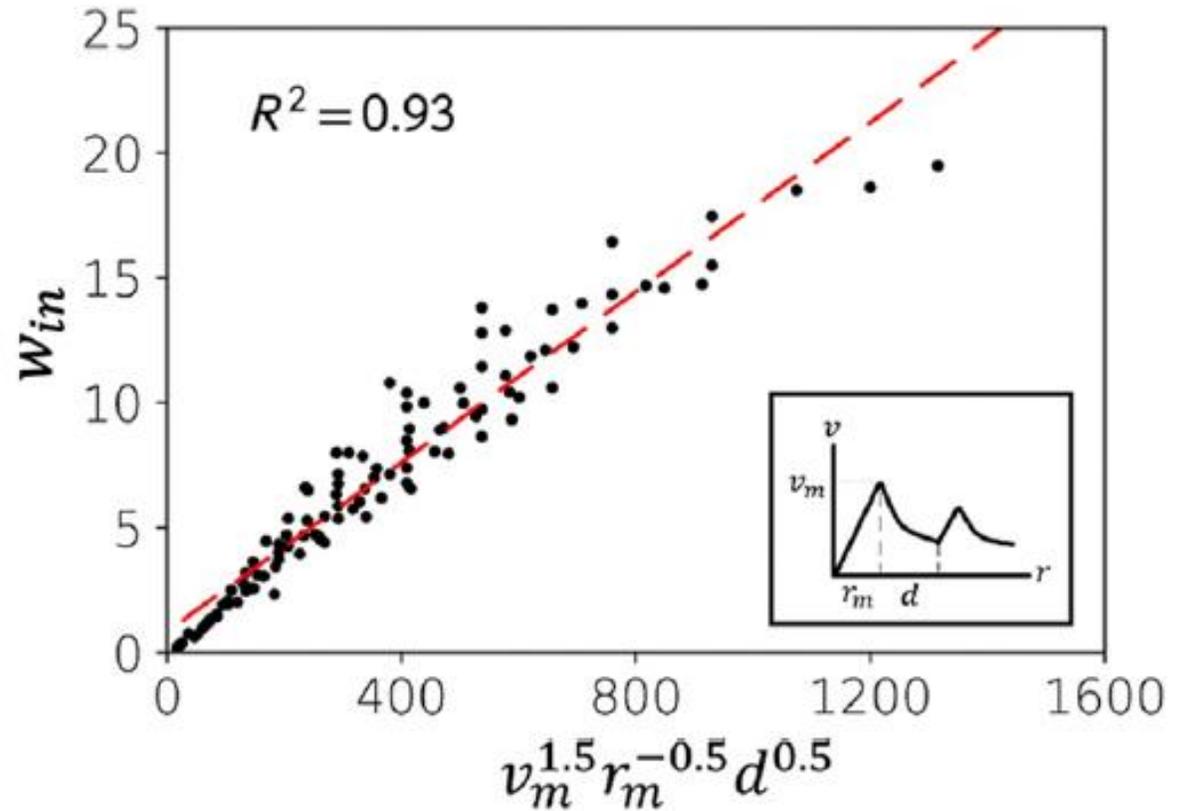
$$u^* \sim w^*$$

$$w^* \sim \sqrt{d^*}$$

In dimensional form:

$$w_{in}/v_m \sim (\zeta d/c)^{0.5} \sim (v_m d/r_m)^{0.5}$$

$$w_{in} \sim v_m^{1.5} r_m^{-0.5} d^{0.5}$$



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# Simulation result

In dimensional form:

$$w^* \sim \sqrt{d^*}$$

$$w_{in}/v_m \sim (\zeta d/c)^{0.5}$$

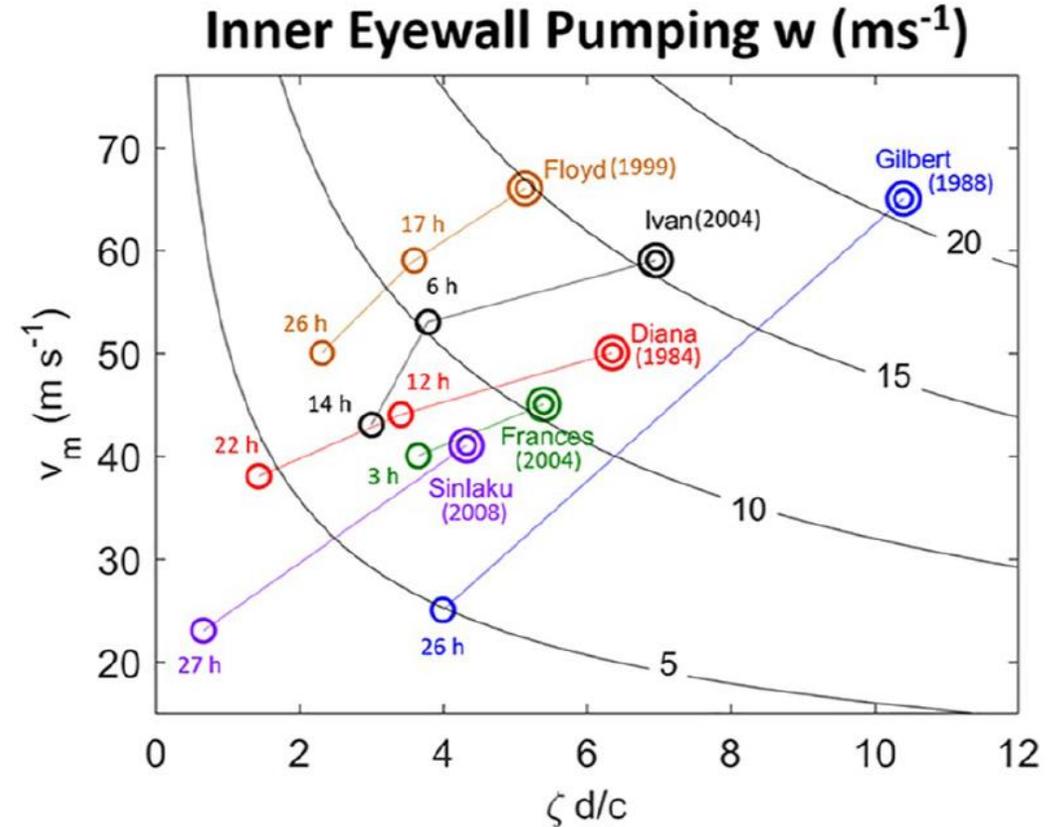
$$w_{in} \sim v_m (\zeta d/c)^{0.5}$$

Points: aircraft observations (tangential wind maximum) based on previous papers.

Both the TC intensity ( $v_m$ ) and dimensionless moat  $\frac{\zeta d}{c}$  decrease during the ERC process.

CE formation:  $v_m > 40 \text{ m/s}$  and  $\frac{\zeta d}{c} > 4$

ERC:  $\frac{\zeta d}{c} < 4$



regression line:  $w_{in} = 0.017 \times v_m^{1.5} r_m^{-0.5} d^{0.5} + 0.85$

$$R^2 = 0.93$$

Only experiments satisfy  $\sqrt{v_m/d} > 0.03$  is considered

# Observation

In dimensional form:

$$w^* \sim \sqrt{d^*}$$

$$w_{in}/v_m \sim (\zeta d/c)^{0.5}$$

$$w_{in} \sim v_m (\zeta d/c)^{0.5}$$

Points: aircraft observations (tangential wind maximum) based on previous papers.

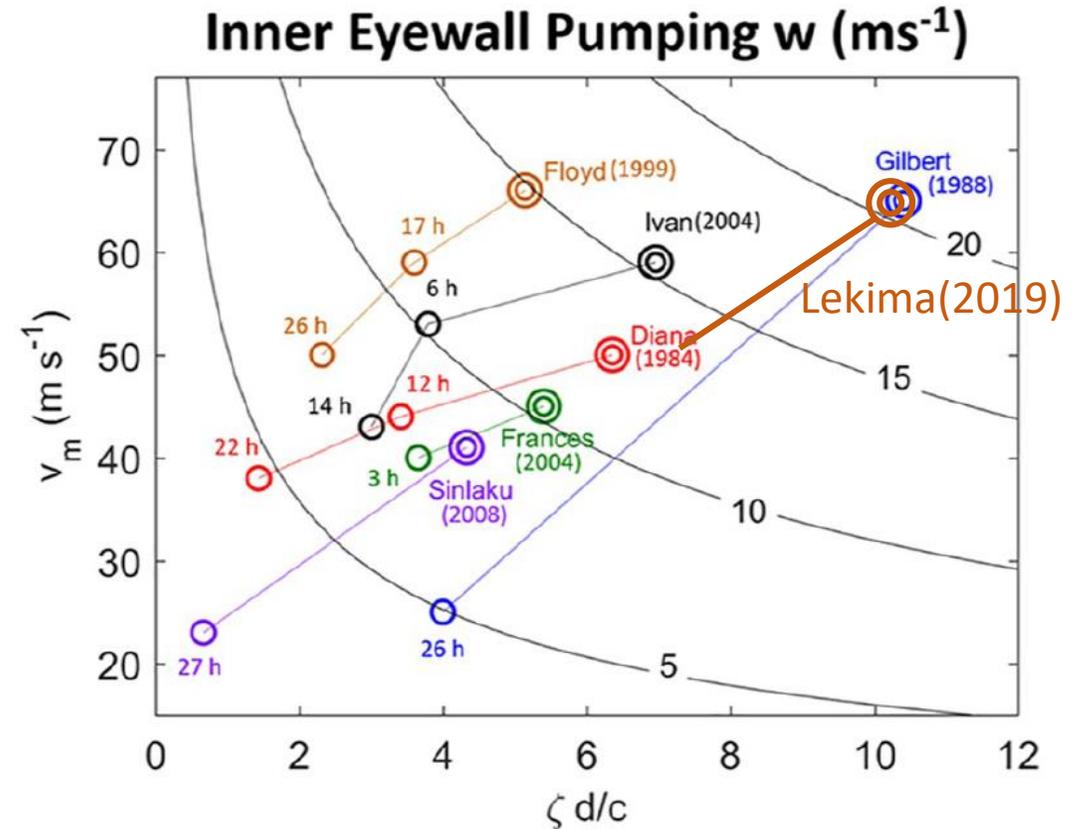
CE formation:  $v_m > 40 \text{ m/s}$  and  $\frac{\zeta d}{c} > 4$

ERC:  $\frac{\zeta d}{c} < 4$

TC Lekima (2019):

CE formation:  $v_m = 65 \text{ m/s}$ ,  $\frac{\zeta d}{c} = 10$

ERC:  $v_m = 50 \text{ m/s}$ ,  $\frac{\zeta d}{c} = 7$



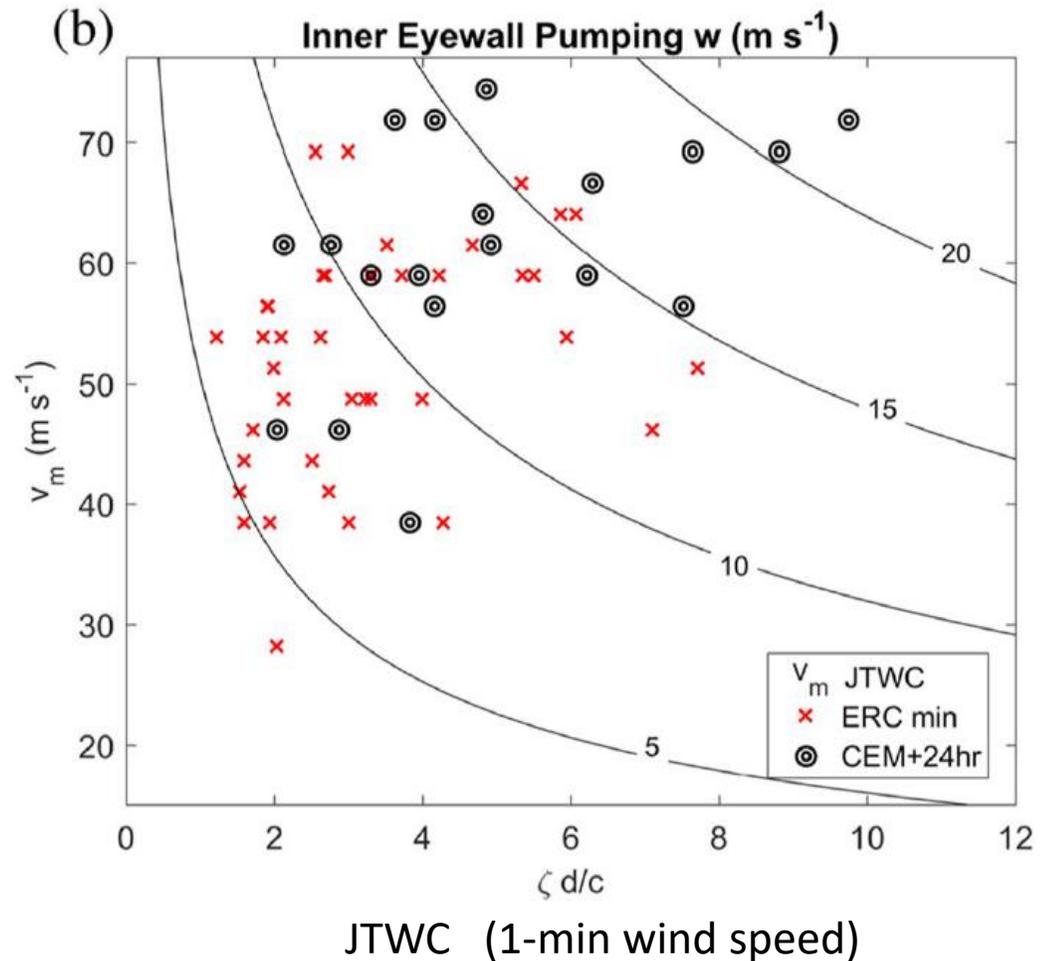
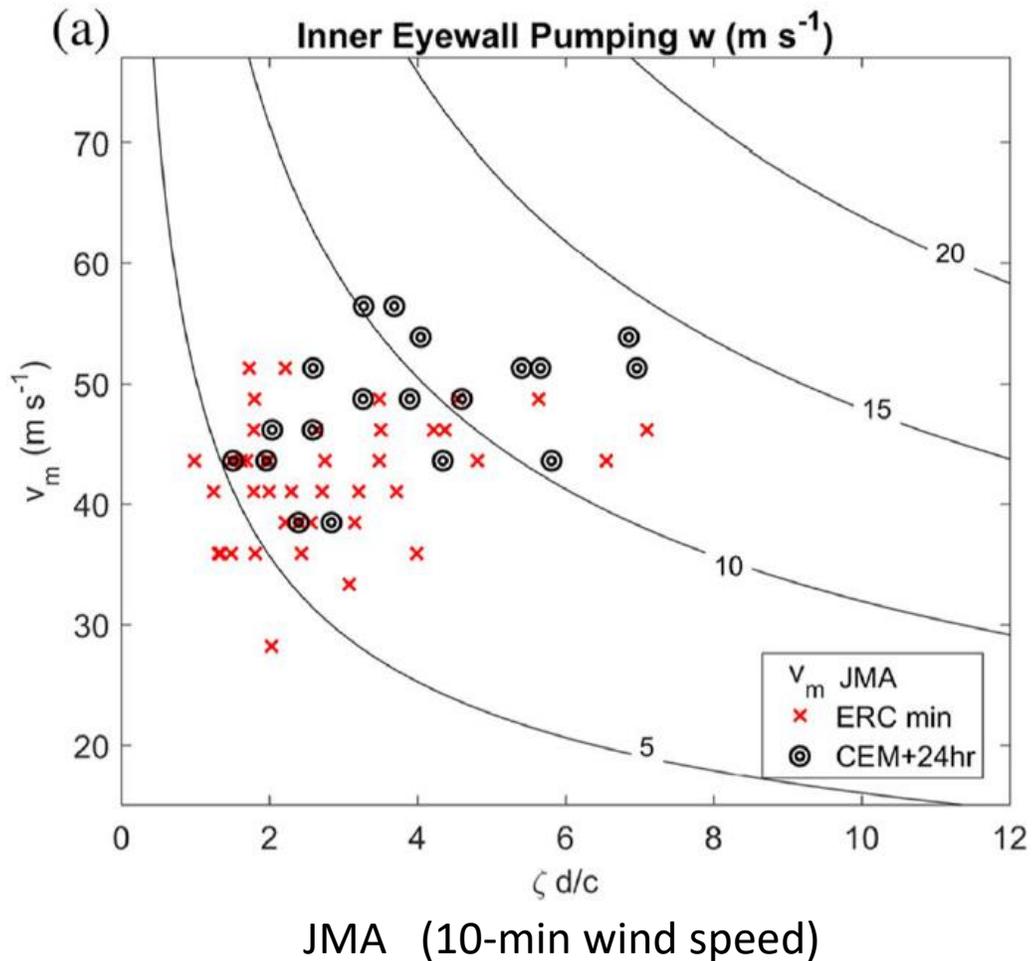
regression line:  $w_{in} = 0.017 \times v_m^{1.5} r_m^{-0.5} d^{0.5} + 0.85$

$$R^2 = 0.93$$

Only experiments satisfy  $\sqrt{v_m/d} > 0.03$  is considered

# Microwave satellite observation of TCs in WNP during 1997-2014

  $CE < 20 \text{ hr}$   
  $CE > 20 \text{ hr}$



# Summary

- Long-lived CE tends to have larger moat size and outer eyewall width. The WNP has far more long-lived CE than in ATL and ENP.
- Results of SBL model:
  - The inflow reduced to 0 when it passed through the outer eyewall and reaccelerated in the moat by PGF.
  - The IEP is not sensitive to parameters of outer eyewall.
  - Both the large moat size and large PGF can enhance inflow to a large IEP.
  - The scaling law of IEP:  $w_{in} \sim v_m \sqrt{\zeta d/c} \sim v_m^{1.5} r_m^{-0.5} d^{0.5}$
- Phase diagram:
  - IEP is a function of  $v_m$  and  $d^*$ .
  - CE forms when  $v_m > 40$  m/s and  $d^* > 4$ .
  - ERC is the process that reduces both intensity ( $v_m$ ) and dimensionless moat ( $d^*$ ) and leads to demise the IEP and inner eyewall.