Water Budget of Tropical Cyclones

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References

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Horizontal Precipitation Structure of TCs



Houze 2010 Willoughby 1988

Vertical Precipitation Structure of TCs



Palmen and Newton 1969

 Continuity equation for condensed water and ice is written as:

$$\rho(c-e) = \frac{\partial}{\partial t} (\rho q_w) + \nabla \cdot \rho \nabla q_w - K_H \rho \nabla_H^2 q_w$$
$$- K_z \frac{\partial}{\partial z} \left(\rho \frac{\partial q_w}{\partial z} \right) - \frac{\partial}{\partial z} (\rho V_T q_p), \quad (1)$$

where q_w is the total mixing ratio for clouds and precipitation;
q_p is the mixing ratio for precipitation;
V_T is the terminal velocity for precipitation;
P is the air density;
c is the condensation plus deposition;
e is the evaporation plus sublimation

Gamache, Houze, and Marks (1993)

 Bulk condensation (plus deposition) is defined as the volume integral of local condensation (plus deposition):

$$C = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\max}} \delta\rho(c-e) r dr d\theta dz, \qquad (2)$$

 Bulk evaporation (plus sublimation) is defined as the volume integral of local evaporation (plus sublimation):

$$E = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\max}} (\delta - 1)\rho(c - e) r dr d\theta dz.$$
(3)

• The volume integral of the total flux divergence of clouds and precipitation is:

$$T_{H} = \int_{z_{b}}^{z_{i}} \int_{0}^{2\pi} \int_{0}^{r_{\max}} \nabla_{H} \cdot \left[\rho \mathbf{V}_{H}(q_{c}+q_{p})\right] r dr d\theta dz, \quad (4)$$

• Using the divergence theorem, this term can also be written as:

$$T_{H} = r_{\max} \int_{z_{b}}^{z_{f}} \int_{0}^{2\pi} \rho(r_{\max}, \theta, z) V_{r}(r_{\max}, \theta, z)$$
$$\times [q_{c}(r_{\max}, \theta, z) + q_{p}(r_{\max}, \theta, z)] d\theta dz. \quad (5)$$

where T_H is the horizontal transport across the boundary

• By the divergence theorem, the vertical integral of vertical divergence of cloud flux is:

$$T_{zc} = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{max}} \frac{\partial}{\partial z} (\rho w q_c) r dr d\theta dz$$

= $\int_0^{2\pi} \int_0^{r_{max}} \rho(r, \theta, z_T) w(r, \theta, z_T) q_c(r, \theta, z_T) r dr d\theta$
 $- \int_0^{2\pi} \int_0^{r_{max}} \rho(r, \theta, z_B) w(r, \theta, z_B) q_c(r, \theta, z_B) r dr d\theta$
= $T_{zcT} + T_{zcB}$, (6)

• which is equal to the outward advection of cloud water through the top boundary (T_{zcT}) and bottom boundary (T_{zcB}) .

 The net mass of precipitation exiting the budget volume through the top and bottom boundaries is R_{net}:

$$R_{\text{net}} = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\text{max}}} \frac{\partial}{\partial z} \left[\rho(w - V_T) q_p \right] r dr d\theta dz$$

$$= \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho(r, \theta, z_t) \left[w(r, \theta, z_t) - V_T(r, \theta, z_t) \right] q_p(r, \theta, z_t) r dr d\theta$$

$$- \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho(r, \theta, z_b) \left[w(r, \theta, z_b) - V_T(r, \theta, z_b) \right] q_p(r, \theta, z_b) r dr d\theta = -R_T + R_B, \qquad (7)$$

- where R_T is the mass of precipitation falling into the top of the budget volume, and R_B is the rain falling out of the bottom of the budget volume.
- Note that if the bottom is near surface, R_B is approximately the area-averaged rainfall rate.

• The volume integral of horizontal diffusion is:

$$D_{H} = -\int_{z_{b}}^{z_{t}} \int_{0}^{2\pi} \int_{0}^{r_{\max}} \rho K_{H} \nabla \cdot \nabla_{H} (q_{c} + q_{p}) r dr d\theta dz$$

= $-K_{H} r_{\max} \int_{z_{b}}^{z_{t}} \int_{0}^{2\pi} \rho (r_{\max}, \theta, z) \frac{\partial}{\partial r} [q_{c}(r_{\max}, \theta, z) + q_{p}(r_{\max}, \theta, z)] d\theta dz.$

 where D_H is positive when the net diffusion is outward from the volume.

• The volume integral of vertical diffusion is:

$$\begin{split} D_{z} &= -\int_{z_{b}}^{z_{t}} \int_{0}^{2\pi} \int_{0}^{r_{\max}} \frac{\partial}{\partial z} \left(\rho K_{z} \frac{\partial}{\partial z} \left[q_{c} + q_{p} \right] \right) r dr d\theta dz \\ &= -\rho(z_{t}) K_{z} \int_{0}^{2\pi} \int_{0}^{r_{\max}} \frac{\partial}{\partial z} \left[q_{c}(r, \theta, z_{t}) + q_{p}(r, \theta, z_{t}) \right] r dr d\theta \\ &+ \rho(z_{b}) K_{z} \int_{0}^{2\pi} \int_{0}^{r_{\max}} \frac{\partial}{\partial z} \left[q_{c}(r, \theta, z_{b}) + q_{p}(r, \theta, z_{b}) \right] r dr d\theta = D_{T} + D_{B}, \end{split}$$

where D_T (positive upward) is the diffusion out of the top surface and D_B (positive downward) is the diffusion out of the bottom surface.

• The steady-state bulk water budget can be expressed as:

$$\rho(c-e) = \frac{\partial}{\partial t} (\rho q_w) + \nabla \cdot \rho \nabla q_w - K_H \rho \nabla_H^2 q_w$$
$$- K_z \frac{\partial}{\partial z} \left(\rho \frac{\partial q_w}{\partial z} \right) - \frac{\partial}{\partial z} (\rho V_T q_p), \quad (1)$$

• or

$$C + R_T = E + T_H + T_{zcT} + T_{zcB} + R_B + D_H + D_T + D_B, \quad (10)$$



$$C + R_T = E + T_H + T_{zcT} + T_{zcB} + R_B + D_H + D_T + D_B, \quad (10)$$



 $\mathbf{C} + \mathbf{R}_{\mathrm{T}} = \mathbf{E} + \mathbf{T}_{\mathrm{H}} + \mathbf{T}_{\mathrm{ZCT}} + \mathbf{T}_{\mathrm{ZCB}} + \mathbf{R}_{\mathrm{B}} + \mathbf{D}_{\mathrm{H}} + \mathbf{D}_{\mathrm{B}} + \mathbf{D}_{\mathrm{T}} + \mathbf{S}$

FIG. 1. Schematic of hurricane bulk water budget. The budget volume is a cylinder. One sector is cut away to show the regions in which the various processes occur. Terms are defined in section 2.

Gamache, Houze, and Marks (1993)

Hurricane Nobert (1984) --- calculations from radar data analysis

Bulk water budgets of Hurricane Nobert (1984) from two radar-retrieval methods



FIG. 17. Bulk water budgets by quadrant. C, E, R, and R_T are bulk condensation, evaporation, rain falling through the bottom, and rain falling through the top, respectively. Units are 10⁹ kilograms per hour. Arrows indicate bulk advection through the indicated boundary. The budgets of methods 1a and 2b are shown.

Gamache, Houze, and Marks (1993)

Full storm budgets of Hurricane Nobert (1984) from two radar-retrieval methods

TABLE 2. Full storm budgets for methods 1 and 2 and method 2 applied to the axisymmetric wind field. Terms in the bulk budget described by (10) are labeled in column 1 as they are in (10). Units are 10^9 kg h^{-1} .

Budget term	Method 1 (10 ⁹ kg h ⁻¹)	Method 2 (10 ⁹ kg h ⁻¹)	Axisymmetry (10 ⁹ kg h ⁻¹)
Condensation (C)	113.61	123.26	51.76
Evaporation (E)	71.76	57.73	16.20
Net condensation	(41.86)	(65.53)	35.56
Radial cloud advection	1.95	0.73	0.15
Radial precipitation			
advection	1.58	-0.82	1.27
Radial water advection			
(T_H)	3.53	-0.09	1.42
Bottom cloud			
advection (T_{zcB})	-0.37	-0.43	-0.03
Top cloud advection			
(T_{RT})	-0.31	0.28	-0.03
Radial water diffusion			
(D_H)		-1.28	1.42
Bottom water diffusion			
(D_B)		3.02	1.42
Top water diffusion			
(D_T)	\frown	1.83	1.56
Rain (R _a)	(41.16)	(65.38)	33.27
Top rain (R_T)	1.18	2.32	2.36
Radial vapor			
advection		-13.36	-10.14
Top vapor advection		0.00	0.11
Bottom vapor			
advection		-32.88	-28.59
Radial vapor diffusion		1.50	6.03
Top vapor diffusion		0.43	0.47
Bottom vapor			
diffusion		-21.07	-0.04

Gamache, Houze, and Marks (1993)

Full storm budgets of Hurricane Nobert (1984) from two radar-retrieval methods



Gamache, Houze, and Marks (1993)



Hurricane Bonnie (1998) --- calculations from the model simulation

Braun, S. A., 2006: High-resolution simulation of Hurricane Bonnie (1998). Part II: Water budget. *J. Atmos. Soc.*, **63**, 43–64.

Radar Reflectivity Comparison



Radar Reflectivity CFAD







FIG. 2. (a) Single-level histogram of radar reflectivity at 8 km for 2139 UTC. Histogram bin width is 5 dBZ. (b) Perspective view of frequency by altitude diagram of radar reflectivity for 2139 UTC volume. Heavy line corresponds to single-level histogram at 8 km shown in (a). (c) The 2139 UTC reflectivity CFAD. The bin size is 5 dBZ, and the plot is contoured at intervals of 2.5% of data per dBZ per kilometer with the 5% dBZ⁻¹ km⁻¹ contour highlighted. Horizontal dashed line at 8 km corresponds to data contained in the single-level histogram in (a). The CFAD in (c) is truncated above 14.8 km where the number of available data points were considered too few to be representative of the storm structure. See appendix A for further details.

40 m

2.7 kr



FIG. 3. Shading indicates time-averaged simulated radar reflectivity at (a) 40 m, (b) 2.7 km, (c) 6.8 km, and (d) 12.0 km, with contours drawn at 15, 25, 35, and 45 dBZ (light, medium, dark, and dotted, respectively). Contour overlays in (a), (d) are storm-relative radial velocities

1-h Time Average (24-25 h)



FIG. 4. Shading indicates (a), (c) time-averaged simulated radar reflectivity and (b) total cloud mixing ratio at Y = 0 km (see Fig. 3). Shaded contours are drawn at 10, 20, 30, and 40 dBZ (light, medium, dark, and dotted, respectively) in (a), (c) and at 0.1, 0.2, 0.4, 0.6, and 0.8 g kg⁻¹ (light, medium, medium-dark, dark, and dotted) in (b). Contour overlays in (a), (b) are



FIG. 6. Azimuthally averaged water vapor budget fields showing (a) condensation, (b) horizontal flux divergence, (c) evaporation, (d) vertical flux divergence, (e) net condensation [sum of (a) and (c)], (f) total flux divergence [sum of (b) and (d) and approximately equivalent to the total advection], (g) divergence term, and (h) boundary layer source term. Contour intervals in (a), (e), (f) are $2 \text{ g m}^{-3} \text{ h}^{-1}$, with extra contours at $\pm 1 \text{ g m}^{-3} \text{ h}^{-1}$. Contour values

$$\begin{bmatrix} \overline{\delta u_s} \\ \overline{\partial t} \end{bmatrix} \approx -\begin{bmatrix} c_p \theta_{v0} \frac{\partial \pi}{\partial x} \end{bmatrix} - \begin{bmatrix} \frac{\partial}{\partial x} (\overline{u_s}^2) \end{bmatrix}$$

TEN PGF HMF
$$-\frac{1}{\rho_0} \frac{\partial}{\partial z} S_m - \frac{1}{\rho_0} \frac{\partial}{\partial z} S_e \quad (14)$$

VMF VEF.

The term on the left of (14) is net momentum tendency (TEN), and terms on the right of (14) are the horizontal PGF, horizontal mean-flow flux convergence (HMF), vertical mean-flow flux convergence (VMF), and vertical eddy-flux convergence by standing eddies (VEF), respectively. Note from (12c) that the last two terms in (14)



FIG. 20. Momentum tendencies of large-scale area A by terms in (14)—HMF, VMF, PGF, VEF, and TEN—during the mature stage (t = 10-11 h).

Yang and Houze (1996)

Vertical Velocity CFAD



FIG. 7. Shading indicates the cumulative percentage of the area occupied by updrafts less than the indicated value. Contours are drawn at 10% intervals, with additional contours at 95% and 99%. Thick lines show the cumulative percentage of total condensation occurring in updrafts less than the indicated values. (a) Eyewall region and (b) outer region.



FIG. 8. Azimuthally averaged cloud budget fields showing the (a) cloud sink, (b) horizontal flux divergence, (c) net source, (d) vertical flux divergence, (e) boundary layer source, and (f) added water mass to offset negative mixing ratios. The contour interval in (a) is 2 g m⁻³ h⁻¹, with an extra contour at -1 g m⁻³ h⁻¹. Contour values



FIG. 9. Azimuthally averaged precipitation source terms showing (a), (c), (e) sources and (b), (d), (f) sinks for rain, graupel, and snow, respectively. The contour interval is 2 g m⁻³ h⁻¹ with extra contours drawn at ± 1 g m⁻³



FIG. 10. Azimuthally averaged precipitation budget fields showing the (a) net microphysical source, (b) horizontal flux divergence, (c) precipitation fallout and vertical flux divergence, and (d) added water mass to offset negative mixing ratios. The contour interval in (a)–(c) is $2 g m^{-3} h^{-1}$, with extra contours at $\pm 1 g m^{-3} h^{-1}$. Contour





FIG. 12. Vertically integrated source terms for (a) total rain source, (b) warm rain source (rain source plus graupel sink), (c) cold rain source (graupel sink), and (d) graupel source.

- The ocean source of vapor in eyewall is very small relative to the condensation and inward transport of vapor, indicating that many observation studies generally overestimated the role of ocean source by underestimating the radial transport of moisture in the lowest 500 m.
- This finding emphasizes the importance of the lowest 500 m of the hurricane in providing the bulk of water supply to eyewall, while the airborne Doppler radars and aircrafts usually have difficulty in observing the inflow in the lowest 500 m.
- For a mature TC, the azimuthally averaged cloud amount is consumed as fast as it is produced; Cloud liquid water often peaks within the melting layer where cooling by melting enhances condensation.

Conclusions (more)

- In the eyewall, most of the condensation occurs within convective towers while in outer regions condensation results from a mix of stratiform (primarily) and convective (secondary) precipitation processes.
- The precipitation budget is dominated by production and fallout with little precipitation from the eyewall being transported outward into the surrounding precipitation area.
- Much of the mass that is transported outward from the eyewall is in the form of small ice particles at upper levels that provide seeds for additional particle growth by deposition and aggregation.

Typhoon Nari (2001) --- calculations from the model simulation

Yang, M.-J., S. A. Braun, and D.-S. Chen, 2011: Water budget of Typhoon Nari (2001). *Mon. Wea. Rev.*, in press.



3-day rainfall (09/16~09/18)

OBS

6km MM5

2km MM5



• Water vapor budget: q_v

$$\frac{\partial q_{v}}{\partial t} = -\nabla \cdot (q_{v} \mathbf{V}') - \frac{\partial (q_{v} w)}{\partial z} + q_{v} \left(\nabla \cdot \mathbf{V}' + \frac{\partial w}{\partial z} \right) - C + E + B_{v} + D_{v} + Resd_{v}$$

is the storm-relative horizontal air motion; where ${\mathcal W}$ is the vertical air motion: \boldsymbol{C} is the condensation and deposition; Eis the evaporation and sublimation; B_{v} is contribution by PBL and turbulence; is the numerical diffusion term for vapor; D_v $Resd_v$ is the residual term for vapor. Braun (2006) and Yang et al. (201

• Cloud budget: $q_c = q_w + q_i$

$$\frac{\partial q_c}{\partial t} = -\nabla \cdot (q_c \mathbf{V'}) - \frac{\partial (q_c w)}{\partial z} + q_c \left(\nabla \cdot \mathbf{V'} + \frac{\partial w}{\partial z} \right) + Q_{c+} - Q_{c-} + B_c + D_c + Resd_c$$

where

- Q_{c+} is the microphysical source term;
- Q_{c-} is the microphysical sink term;
 - B_c is the contribution by the PBL and turbulence;
 - $\overline{\mathbf{D}_{c}}$ is the numerical diffusion term for clouds;
- *Resd* is the residual term for clouds

Braun (2006) and Yang et al. (2011)

• Precipitation budget: $q_p = q_r + q_s + q_g$

$$\frac{\partial q_{p}}{\partial t} = -\nabla \cdot (q_{p}\mathbf{V}') - \frac{\partial (q_{p}w)}{\partial z} + q_{p}\left(\nabla \cdot \mathbf{V}' + \frac{\partial w}{\partial z}\right) + \frac{\partial (q_{p}V_{T})}{\partial z} + Q_{p+} - Q_{p-} + D_{p} + Resd_{p}$$

$$C - E = Q_{c+} - Q_{c-} + Q_{p+} - Q_{p-}$$

 Q_{p} + is the microphysical source term; Q_{p} - is the microphysical sink term; D_{p} is the numerical diffusion term for precipitation; $Resd_{p}$ is the residual term for precipitation; V_{T} is the hydrometeor terminal velocity

Braun (2006) and Yang et al. (2011)



Definition of Average

• Temporal and azimuthal mean is defined as:

$$\overline{[]} = \frac{1}{2\pi(T_2 - T_1)} \int_{T_1}^{T_2} \int_0^{2\pi} \rho[] \partial \lambda \partial t$$

• Time-averaged and vertically integrated amount is defined as:

$$\left[\hat{}\right] = \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} \int_{Z_B}^{Z_T} \rho[] \partial z \partial t$$

Time-averaged, volumetrically integrated amount is defined as

$$= \int_{Z_B}^{Z_T} \int_{R_1}^{R_2} \overline{[]} 2\pi r \partial r \partial z$$

Typhoon Nari over the Ocean



Oceanic Nari in the Across-Track Cross Section



Axis-symmetric Structure of Nari over Ocean



Typhoon Nari at the Landfall Stage



Nari Structure over Land in along-track direction





Nari Structure over Land in across-track direction



Axis-symmetric Water-Vapor Budget Terms for Oceanic Nari



Water-Vapor Budget Terms for Landfall Nari in across-track direction



Axis-symmetric Liquid/Ice Water Budget Terms for Oceanic Nari



Axis-symmetric Hydrometeor Source/Sink Terms for Oceanic Nari



Liquid/Ice Water Budget Terms for Landfall Nari in along-track direction



Time-averaged & Vertically-integrated Amount of Rain Source/Sink



Time-averaged and vertically-integrated amount of microphysical sources and sinks





• Water vapor budget equation can be written as:

Tend = Cond + Evap + HFC + VFC + Div + Diff + PBL + Resd

Note that

$$\frac{\partial q_{v}}{\partial t} = -\nabla \cdot (q_{v} \mathbf{V}') - \frac{\partial (q_{v} w)}{\partial z} + q_{v} \left(\nabla \cdot \mathbf{V}' + \frac{\partial w}{\partial z} \right) - C + E + B_{v} + D_{v} + Resd_{v}$$

Liquid/Ice water budget equation can be written as:

Tend = Cond + Evap + HFC + VFC + Div + Diff + PBL + P + Resd

Note that

$$\frac{\partial q_{c}}{\partial t} = -\nabla \cdot (q_{c} \mathbf{V}') - \frac{\partial (q_{c} w)}{\partial z} + q_{c} \left(\nabla \cdot \mathbf{V}' + \frac{\partial w}{\partial z} \right) + Q_{c+} - Q_{c-} + B_{c} + D_{c} + Resd_{c} + \frac{\partial q_{p}}{\partial t} = -\nabla \cdot (q_{p} \mathbf{V}') - \frac{\partial (q_{p} w)}{\partial z} + q_{p} \left(\nabla \cdot \mathbf{V}' + \frac{\partial w}{\partial z} \right) + \frac{\partial (q_{p} V_{T})}{\partial z} + Q_{p+} - Q_{p-} + D_{p} + Resd_{p}$$

 $C - E = Q_{c+} - Q_{c-} + Q_{p+} - Q_{p-}$

Water Vapor Budgets during the Oceainc and Landfall Stages



Liquid/Ice Water Budgets during the Oceainc and Landfall Stages



Precipitation Efficiency as a Function of Storm Radius

$$CMPE = \frac{P}{Cond}$$

$$LSPE = \frac{P}{HFP + VFP}$$





- For the vapor budget, while Nari is over the ocean, evaporation from the ocean surface is 11% of the inward horizontal vapor transport within the 150-km radius from the storm center, and the net horizontal vapor convergence into the storm is 88% of the net condensation.
- The ocean source of water vapor in the inner core is a small portion (5.5%) of horizontal vapor import, consistent with previous studies.
- After landfall, Taiwan's steep terrain enhances Nari's secondary circulation significantly; the net horizontal vapor convergence into the storm within 150 km is increased to 122% of the net condensation after landfall.

- For the condensed water budget, summation of precipitation fallout and total flux convergence is largely out of phase with the net microphysical source term, indicating that precipitation particles are falling out as quickly as they are produced.
- Warm rain processes dominate in the eyewall region, while the cold rain processes are comparable to warm rain processes outside of the eyewall.
- After landfall, cold rain processes are further enhanced above the Taiwan terrain and the storm-total condensation within 150 km from the center is increased by 22%.

- Precipitation efficiency, defined by either the large-scale or microphysics prospective, is increased 10–20% over the outer-rainband region after landfall, in agreement with the enhanced surface rainfall over terrain.
- At radii greater than 60 km, the cloud microphysics precipitation efficiency remains a constant value of 67% for the oceanic stage of Nari and 73% for the landfall stage.
- The reason why the precipitation efficiency remains roughly constant at these radii may be that the region outside of the eyewall is dominated by stratiform precipitation with a relatively constant precipitation efficiency.